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Analytical integrability of the Rikitake system

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Abstract. In this paper, we study the first integrals of the Rikitake system

$$\dot{x} = -\mu x + yz, \quad \dot{y} = -\mu y + x(z - a), \quad \dot{z} = 1 - xy,$$

that can be described by formal power series when the system has singular points.

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1. Introduction and statement of the results

The earth's magnetic field has been reversed aperiodically. The reversals of geomagnetic field are explained by a simple two-disk dynamo system, called the Rikitake system:

$$\dot{x} = -\mu x + yz, \quad \dot{y} = -\mu y + x(z - a), \quad \dot{z} = 1 - xy, \tag{1}$$

when a, μ are real parameters and the system has a singular point. This means that $a, \mu \in \mathbb{R}$ with the exception of the case $\mu = 0$ and $a \neq 0$.

Rikitake system has been analyzed by many researchers, from various view points. The chaotic magnetic reversals have been discussed by numerical or analytical approaches, see for instance [1,2,5,6,14]. Mathematically, algebraic and geometric methods have been applied to find first integrals for the Rikitake system or for studying its dynamics, see [7,8,10–12,20].

In fact the formulation of the Rikitake systems is not unique. In this paper we study the formulation given by the 3D differential system (1) with two parameters a and μ introduced in [18] (see also [4]). Previously in [17], another formulation was used given by a different 3D differential system depending on three parameters instead of two. The integrability of this previous formulation of the Rikitake system was studied in [12,20]. Thus in [12], the authors characterize the invariant algebraic surfaces of those Rikitake systems. These surfaces are the main ingredient for applying the Darboux theory of integrability, which was studied in [20]. Also in this last paper, it was studied the existence of analytic first integrals. We recall that in general the Darboux integrability does not imply the analytic one, and that the analytic integrability does not imply the Darboux one, except when the first integral provided by the Darboux theory of integrability is analytic.

The Darboux integrability of the Rikitake system formulated as in (1) was studied in [10]. In spite of these previous studies, the analytic first integrals of the Rikitake system (1) have not been studied. This is the main objective of this paper. Thus, we completely study the analytic integrability of system (1) when $a = 0$. When $a \neq 0$ we show that system (1) has no analytic first integrals in the variables x, y, z and a, but unfortunately we do not know how to study if there are or not analytic first integrals only depending on x, y and z. Therefore, the problem of the analytic integrability of system (1) for a fixed