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ON THE LIMIT CYCLES OF POLYNOMIAL DIFFERENTIAL SYSTEMS WITH HOMOGENEOUS NONLINEARITIES OF DEGREE 3 VIA THE AVERAGING METHOD

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Abstract. We study the limit cycles of a class of cubic polynomial differential systems in the plane and their global shape using the averaging theory. More specifically, we analyze the global shape of the limit cycles which bifurcate: first, from a Hopf bifurcation; second, from periodic orbits of the linear center $\dot{x} = -y$, $\dot{y} = x$; and finally from periodic orbits of the cubic centers $\dot{x} = -yh(x, y)$, $\dot{y} = xh(x, y)$ where $h(x, y) = 0$ is a conic. The perturbation of these systems is made inside the class of cubic polynomial differential systems having non quadratic terms.

Keywords. limit cycles, cubic vector fields, cubic polynomial differential systems, averaging theory.

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1 Introduction and main results

In this paper we deal with the cubic polynomial differential systems (or in what follows simply *cubic systems*) of the form

$$\begin{aligned} \dot{x} = \quad & p_1(x, y) + p_3(x, y), \\ \dot{y} = \quad & q_1(x, y) + q_3(x, y), \end{aligned} \tag{1}$$

where p_i and q_i denote homogeneous polynomials of degree i. So the unique nonlinearities of these systems are homogeneous polynomials of degree 3.

We study the limit cycles of the cubic systems (1) and their global shape using the averaging theory. More specifically, we shall study

(i) the global shape of the limit cycles which born in a Hopf bifurcation at the origin of system (1), see Proposition 1;

(ii) the global shape of the limit cycles of systems (1) which bifurcate from periodic orbits of the linear center $\dot{x} = -y$, $\dot{y} = x$, see Proposition 2; and