Classification of centers, their cyclicity and isochronicity for a class of polynomial differential systems of degree $d \geq 7$ odd

Jaume Llibre Clàudia Valls

Abstract

In this paper we classify the centers, the cyclicity of its Hopf bifurcation and the isochronicity of the polynomial differential systems in $\mathbb{R}^2$ of degree $d \geq 7$ odd that in complex notation $z = x + iy$ can be written as

$$\dot{z} = (\lambda + i)z + (z\bar{z})^{\frac{d-7}{2}}(Az^6\bar{z} + Bz^4\bar{z}^3 + Cz^2\bar{z}^5 + D\bar{z}^7),$$

where $\lambda \in \mathbb{R}$, and $A, B, C, D \in \mathbb{C}$.

1 Introduction and statement of the main results

Probably the two main problems in the qualitative theory of real planar polynomial differential systems are the determination of limit cycles and the center–focus problem; i.e. to distinguish when a singular point is either a focus or a center. The notion of *center* goes back to Poincaré in [16]. He defined it for a vector field on the real plane; i.e. a singular point surrounded by a neighborhood filled with periodic orbits with the unique exception of the singular point. This paper deals with the center–focus problem for a class of polynomial differential systems of degree $d \geq 7$ odd. Note that there are few results on families of centers of polynomial differential systems of arbitrary degree.

The classification of centers for the polynomial differential systems started with the quadratic ones with the works of Dulac [6], Kapteyn [10, 11], Bautin [2], Żołądek [18], ... see Schlomiuk [17] for an update on the quadratic centers.

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