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Classification of the centers and their isochronicity for a class of polynomial differential systems of arbitrary degree

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Abstract

In this paper we classify the centers localized at the origin of coordinates, and their isochronicity for the polynomial differential systems in $\mathbb{R}^2$ of degree d that in complex notation $z = x + iy$ can be written as

$$\dot{z} = (\lambda + i)z + Az^{(d-n+1)/2}\bar{z}^{(d+n-1)/2} + Bz^{(d+n+1)/2}\bar{z}^{(d-n-1)/2} + Cz^{(d+1)/2}\bar{z}^{(d-1)/2} + Dz^{(d-(2+j)n+1)/2}\bar{z}^{(d+(2+j)n-1)/2},$$

where j is either 0 or 1. If $j = 0$ then $d \geqslant 5$ is an odd integer and n is an even integer satisfying $2 \leqslant n \leqslant (d+1)/2$. If $j = 1$ then $d \geqslant 3$ is an integer and n is an integer with converse parity with d and satisfying $0 < n \leqslant [(d+1)/3]$ where $[\cdot]$ denotes the integer part function. Furthermore $\lambda \in \mathbb{R}$ and $A, B, C, D \in \mathbb{C}$. Note that if $d = 3$ and $j = 0$, we are obtaining the generalization of the polynomial differential systems with cubic homogeneous nonlinearities studied in K.E. Malkin (1964) [17], N.I. Vulpe and K.S. Sibirskii (1988) [25], J. Llibre and C. Valls (2009) [15], and if $d = 2$, $j = 1$ and $C = 0$, we are also obtaining as a particular case the quadratic polynomial differential systems studied in N.N. Bautin (1952) [2], H. Zoladek (1994) [26]. So the class of polynomial differential systems here studied is very general having arbitrary degree and containing the two more relevant subclasses in the history of the center problem for polynomial differential equations.

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