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HOPF BIFURCATION FOR SOME ANALYTIC DIFFERENTIAL SYSTEMS IN $\mathbb{R}^3$ VIA AVERAGING THEORY

Jaume Llibre

Departament de Matemàtiques, Universitat Autònoma de Barcelona
08193 Bellaterra, Barcelona, Catalonia, Spain

Clàudia Valls

Departamento de Matemática, Instituto Superior Técnico
Av. Rovisco Pais 1049-001, Lisboa, Portugal

Abstract. We study the Hopf bifurcation from the singular point with eigenvalues $a\,\varepsilon \pm b\,i$ and $c\,\varepsilon$ located at the origen of an analytic differential system of the form $\dot{\mathbf{x}} = f(\mathbf{x})$, where $\mathbf{x} \in \mathbb{R}^3$. Under convenient assumptions we prove that the Hopf bifurcation can produce 1, 2 or 3 limit cycles. We also characterize the stability of these limit cycles. The main tool for proving these results is the averaging theory of first and second order.

1. **Introduction.** The main goal of this work is to study the Hopf bifurcation in analytic differential systems in $\mathbb{R}^3$ via averaging theory. In fact the results obtained can be extended easily to C^4 differential systems in $\mathbb{R}^3$ but in order to simplify the notation we will present them for analytic differential systems.

More precisely, we investigate the Hopf bifurcation at the singular point located at the origin for an analytic differential systems in $\mathbb{R}^3$ of the form

$$\begin{aligned}
\dot{U} &= \varepsilon a U - bV + \sum_{l+j+k\geq n} A_{ijk} U^l V^j W^k,\\
\dot{V} &= bU + \varepsilon a V + \sum_{l+j+k\geq n} B_{ijk} U^l V^j W^k,\\
\dot{W} &= \qquad\quad \varepsilon c W + \sum_{l+j+k\geq n} C_{ijk} U^l V^j W^k,
\end{aligned} \tag{1}$$

where $n = 2, 3$ and ε is a small parameter. Note that the linear part of this system at the singular point located at the origin $(0, 0, 0)$ has eigenvalues $a\,\varepsilon \pm b\,i$ and $c\,\varepsilon$. So for $\varepsilon = 0$ the eigenvalues are $\pm bi$ and 0, consequently we are studying a kind of zero–Hopf bifurcation.

For $n = 2$ the Hopf bifurcation of system (1) was studied in [2] were the authors obtained the following result.

Theorem 1.1 (First order Hopf bifurcation theorem for $n = 2$)**.** *We define the constants*

$$F = A_{101}+B_{011}, G = C_{020}+C_{200}, D = c(cF-4aC_{002}), E = D^2+8ab^2F(cF-2aC_{002}).$$

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