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On the C^1 non-integrability of differential systems via periodic orbits

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We go back to the results of Poincaré [Poincare, H (1891) Sur lintegration des equations differentielles du premier ordre et du premier degre I and II, *Rendiconti del circolo matematico di Palermo* **5**, 161–191] on the multipliers of a periodic orbit for proving the C^1 non-integrability of differential systems. We apply these results to Lorenz, Rossler and Michelson systems, among others.

Key words: $\boldsymbol{C}^1$ integrability, differential systems, periodic orbits

1 Introduction and statements of main results

In these last years the Ziglin and the Morales–Ramis theories have been used for studying the non-meromorphic integrability of an autonomous differential system. In some sense the Ziglin theory is a continuation of Kovalevskaya's ideas used for studying the integrability of a rigid body because it relates the non-integrability of the considered system with the behaviour of some of its non-equilibrium solutions as a function of complex time using the monodromy group of their variational equations. Ziglin's theory was extended to the so-called Morales–Ramis' theory, which replaces the study of the monodromy group of variational equations by the study of the Galois differential group, which is easier to analyse (see [8] for more details and the references therein). But like the Ziglin theory, the Morales–Ramis theory can only study the non-existence of meromorphic first integrals.

Kovalevskaya's ideas and consequently the Ziglin and the Morales–Ramis theories go back to Poincaré's results (see Arnold [1]), who used the multipliers of the monodromy group of variational equations associated to periodic orbits for studying the non-integrability of autonomous differential systems. The main difficulty in applying Poincaré's non-integrability method to a given autonomous differential system is to find for such an equation periodic orbits having multipliers different from 1.

It seems that Poincaré's this result was forgotten by the mathematical community until modern Russian mathematicians (specially Kozlov) wrote on it (see [1, 10]).