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On the C^1 non-integrability of the Belousov–Zhabotinskii system

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ABSTRACT

We use the stability or instability of the singular points together with the results of Poincaré on the multipliers of a periodic orbit for studying the C^1 non-integrability of the Belousov–Zhabotinskii system.

1. Introduction and statements of the main results

The Belousov–Zhabotinskii system (see [1]):

$$\dot{x} = s(x + y - qx^2 - xy), \qquad \dot{y} = s^{-1}(-y + fz - xy), \qquad \dot{z} = w(x - z), \tag{1}$$

is one of the most interesting and best understood dynamical oscillators, where x, y, and z are real variables, and s, f, q, and w are real parameters and $s \neq 0$. This system has been intensively investigated as a dynamical system (see for instance [2–7]). Here the dot denotes derivative with respect to time t.

In MathSciNet, there are now 265 published papers with some relation with the Belousov–Zhabotinskii system but very few is known about the integrability or non-integrability of this system.

A global analytic first integral or simply in what follows *an analytic first integral* is a non-constant analytic function $H: \mathbb{R}^3 \to \mathbb{R}$ whose domain of definition is $\mathbb{R}^3$, and it is constant on the solutions of system (1), i.e.

$$\mathcal{X}H = s(x + y - qx^2 - xy)\frac{\partial H}{\partial x} + s^{-1}(-y + fz - xy)\frac{\partial H}{\partial y} + w(x - z)\frac{\partial H}{\partial z} = 0. \tag{2}$$

In [8], the authors proved the following result concerning the analytic integrability of system (1).

Theorem 1. *The following statements hold for the Belousov–Zhabotinskii system.*

(1) *The unique global analytic first integrals for system (1) with $w = 0$ are of the form $g(z)$ where g is an arbitrary global analytic function.*
(2) *System (1) with $w \neq 0$ has no global analytic first integrals which are also analytic in the parameter w in a neighborhood of $w = 0$.*

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