POLYNOMIAL FIRST INTEGRALS FOR THE CHEN AND LÜ SYSTEMS

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We characterize all the values of the parameters for which the Chen and Lü systems have polynomial first integrals by using weight homogeneous polynomials and the method of characteristics for solving partial differential equations. We improve previous results which were not complete.

Keywords: Darboux integrability; exponential factor; Darboux polynomials; Chen systems; Lü systems.

1. Introduction and Statement of the Main Results

The following real differential system

$$\begin{aligned} \dot{x} &= a(y - x), \\ \dot{y} &= (c - a)x - xz + cy, \\ \dot{z} &= xy - bz, \end{aligned} \tag{1}$$

where $a, b, c \in \mathbb{R}$ are parameters is known as the *Chen system* [Chen & Ueta, 1999]. It exhibits chaotic phenomena which resembles some familiar features from both the Lorenz and the Rössler attractors, for suitable choices of the parameters. Despite its similar structure to the Lorenz system, it is not topologically equivalent. This is why Lü and Chen investigated the real differential system

$$\dot{x} = a(y - x)$$

$$\dot{y} = -xz + cy,$$

$$\dot{z} = xy - bz,$$

(2)

where $a, b, c \in \mathbb{R}$ are parameters, which is now called the $L\ddot{u}$ system [Lü & Chen, 2002]. The Lü system connects the Lorenz system and the Chen system and represents a transition from one to the other. For more details, see [Lü & Chen, 2002]. Moreover, recently Lü and Zhang [2007] and Lü [2009] characterized the invariant algebraic surfaces of the Chen and the Lü systems, respectively. Furthermore, Lü [2007] characterized the Darboux first integrals of the Chen system. These recent years the dynamics of the Chen system has been analyzed from many different points of view. See for instance [Bashkirtseva *et al.*, 2010; Cafagna & Grassi, 2008; Cai *et al.*, 2009; Cao *et al.*, 2008;

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