ON THE POLYNOMIAL INTEGRABILITY OF THE KIRCHOFF'S EQUATIONS

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ABSTRACT. We prove that the Kirchoff's equations either are completely integrable, or have at most four functionally independent polynomial first integrals.

1. Introduction and statement of the main results

Given a system of ordinary differential equations depending on parameters in general is very difficult to recognize for which values of the parameters the equations have first integrals because there are no satisfactory methods to answer this question.

In this paper we study the first integrals of the Kirchoff's differential equations in $\mathbb{R}^6$ depending on six parameters which provide a model of an ellipsoidal rigid body submerged in an ideal fluid, that is, they are derived under the assumption that the rigid body is ellipsoidal and it is submerged in an infinitely large volume of irrotational, incompressible, inviscid fluid that is at rest at infinity. Under circumstances in which viscous effects are small, it is common to use these equations to describe the dominant dynamics of an underwater vehicle which is ellipsoidal.

The Kirchoff's differential equations appeared by first time in the book of Kirchoff [9]. These differential equations have been studied previously for several authors as Kozlov and Onishchenko [11, 12, 13], Holmes, Jenkins and Leonard [8], see moreover the list of references in these articles about this problem. Many different aspects of the motion of the rigid body governed by the Kirchoff's differential equations where studied in those papers, but the question of its completely integrability using only polynomial first integrals was not considered yet. Here we provide an answer to this question.

The motion of the rigid body has been considered from many other different points of view, for instance many people study the motion of the rigid body fixed at a point, then we get the classical Suslov [14], Chaplygin [3] and Veselov–Veselova [15] problems for the rigid body or generalizations of these problems by Dragovic, Gajic and Jovanovic [6], and many other models non–considered in this work, see for more details on the rigid body the book of Kozlov [10].

Let $p = (p_1, p_2, p_3)$ and $\pi = (\pi_1, \pi_2, \pi_3)$ be the linear and angular momentum vectors. Choosing the axes of the body-fixed frame to coincide with the principal axis of the ellipsoid, this yields that the so-called *added mass matrix* is diagonal, i.e. $M = \operatorname{diag}(m_1, m_2, m_3)$ and the so-called *added inertia matrix* is diagonal, i.e. $I = \operatorname{diag}(I_1, I_2, I_3)$. We recall that each mass and inertia term is the sum of a component due to the body and a component due to the fluid.

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