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LIOUVILLIAN AND ANALYTIC FIRST INTEGRALS FOR THE BRUSSELATOR SYSTEM

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We characterize the Liouvillian and analytic first integrals for the polynomial differential systems of the form $x' = a - (b+1)x + x^2y$, $y' = bx - x^2y$, with $a, b \in \mathbb{R}$, called the Brusselator differential systems.

Keywords: Darboux polynomial; exponential factor; Liouvillian first integral; analytic first integral; Brusselator differential systems.

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1. Introduction and Statement of the Main Results

One of the more classical problems in the qualitative theory of planar differential systems depending on parameters is to characterize the existence or not of first integrals.

We consider the system

$$x' = a - (b+1)x + x^2y, \quad y' = bx - x^2y \tag{1}$$

that we call the *Brusselator*, where x and y are complex variables and the prime denotes derivative with respect to the time t, which can be either real or complex. Such differential systems appear in several branches of the sciences, mainly in chemistry since it studies a certain chemical reaction (see [7, 9, 16] for details).

Let $U \subset \mathbb{C}^2$ be an open and dense set in $\mathbb{C}^2$. We say that the non-constant function $H\colon \mathbb{C}^2 \to \mathbb{C}$ is a first integral of the polynomial vector field χ on U, if $H(x(t),$