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On the number of limit cycles of a class of polynomial differential systems

By Jaume Llibre[1,*] and Clàudia Valls[2]

[1] Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain
[2] Departamento de Matemática, Instituto Superior Técnico, 1049–001 Lisboa, Portugal

We study the number of limit cycles of polynomial differential systems of the form

$$\dot{x} = y - g_1(x) - f_1(x)y \quad \text{and} \quad \dot{y} = -x - g_2(x) - f_2(x)y,$$

where g_1, f_1, g_2 and f_2 are polynomials of a given degree. Note that when $g_1(x) = f_1(x) = 0$, we obtain the generalized polynomial Liénard differential systems. We provide an accurate upper bound of the maximum number of limit cycles that the above system can have bifurcating from the periodic orbits of the linear centre $\dot{x} = y$, $\dot{y} = -x$ using the averaging theory of first and second order.

Keywords: limit cycles; polynomial differential systems; Liénard differential systems

1. Introduction

The second part of the 16th Hilbert problem wants to find an upper bound on the maximum number of limit cycles that a polynomial vector field of a fixed degree can have. In this paper, we will try to give a partial answer to this problem for the class of polynomial differential systems given by

$$\dot{x} = y - g_1(x) - f_1(x)y \quad \text{and} \quad \dot{y} = -x - g_2(x) - f_2(x)y. \tag{1.1}$$

Note that when $g_1(x) = f_1(x) = 0$ coincides with generalized polynomial Liénard differential systems. The classical polynomial Liénard differential systems are

$$\dot{x} = y \quad \text{and} \quad \dot{y} = -x - f(x)y, \tag{1.2}$$

where $f(x)$ is a polynomial in the variable x of degree n. For these systems, Lins *et al.* (1977) stated the conjecture that if $f(x)$ has degree $n \geq 1$, then system (1.2) has at most $[n/2]$ limit cycles. They proved this conjecture for $n = 1, 2$. Recently, the conjecture has also been proved for $n = 3$, see Li & Llibre (2012). For $n \geq 5$, Dumortier *et al.* (2007) and De Maesschalck & Dumortier (2011) showed that the conjecture is not true for $n \geq 5$. In short, at this moment, the conjecture is only open for $n = 4$.

*Author for correspondence (jllibre@mat.uab.cat).

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