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Integrability in nonlinear biomathematical models

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ABSTRACT

We study the integrability of two biomathematical models described by quadratic polynomial differential systems in the plane. These two models can be divided into six families of differential systems. For five of these families we classify all the systems which are Darboux integrable or globally analytic integrable.

1. Introduction and statement of the main results

The nonlinear ordinary differential equations appear in many branches of applied mathematics, physics and, in general, in applied sciences. For a differential system defined on an open subset U of the plane the existence of a first integral determines completely its phase portrait. Since for such differential systems the notion of integrability is based on the existence of a first integral the following natural question arises: *Given a differential system on an open subset U of the plane, that eventually can be the whole plane, how to recognize if this differential system has a first integral*?

In this paper, we study the integrability (i.e. the existence of first integrals) in two biological models. The first is due to Swihart et al. [1] in which the behavior of a predator–prey system with different patches is studied. The second model due to Hadeler and Castillo-Chavez [2] studies the homosexual cohorts in which the effectiveness of education is examined.

These two models were analyzed using the Painlevé property by Meletlidou and Leach [3], and they found some of the systems which can be integrated. In their study they divide the first model into the following three differential systems:

$$\begin{array}{lll} \text{(I)} & x' = x(1-x-by), & y' = y(a-y+cx), \\ \text{(II)} & x' = x(-x-by), & y' = y(1-y+cx), \\ \text{(III)} & x' = x(-x-by), & y' = y(-y+cx), \end{array}$$

and the second model again into the next three differential systems:

$$\begin{array}{lll} \text{(IV)} & x' = -x+ay-bxy, & y' = y(c+x-y), \\ \text{(V)} & x' = -x+y-xy, & y' = y(c-y), \\ \text{(VI)} & x' = -x+ay-xy, & y' = y(c+x), \end{array}$$

where $a, b, c \in \mathbb{R}$ and the prime indicates the derivative with respect to the time.

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