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ON THE NUMBER OF LIMIT CYCLES FOR A GENERALIZATION OF LIÉNARD POLYNOMIAL DIFFERENTIAL SYSTEMS

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We study the number of limit cycles of the polynomial differential systems of the form

$$\dot{x} = y - g_1(x), \quad \dot{y} = -x - g_2(x) - f(x)y,$$

where $g_1(x) = \varepsilon g_{11}(x) + \varepsilon^2 g_{12}(x) + \varepsilon^3 g_{13}(x)$, $g_2(x) = \varepsilon g_{21}(x) + \varepsilon^2 g_{22}(x) + \varepsilon^3 g_{23}(x)$ and $f(x) = \varepsilon f_1(x) + \varepsilon^2 f_2(x) + \varepsilon^3 f_3(x)$ where g_{1i}, g_{2i}, f_{2i} have degree k, m and n respectively for each $i = 1, 2, 3$, and ε is a small parameter. Note that when $g_1(x) = 0$ we obtain the generalized Liénard polynomial differential systems. We provide an upper bound of the maximum number of limit cycles that the previous differential system can have bifurcating from the periodic orbits of the linear center $\dot{x} = y$, $\dot{y} = -x$ using the averaging theory of third order.

Keywords: Limit cycles; generalized Liénard polynomial differential systems; averaging theory.

1. Introduction and Statement of the Main Results

The second part of the 16th Hilbert's problem searches for an upper bound for the maximum number of limit cycles that a polynomial vector field of a fixed degree can have. In this paper we will try to give a partial answer to this problem for the class of polynomial differential systems

$$\dot{x} = y - g_1(x), \quad \dot{y} = -x - g_2(x) - f(x)y, \tag{1}$$

where

$$g_1(x) = \varepsilon g_{11}(x) + \varepsilon^2 g_{12}(x) + \varepsilon^3 g_{13}(x),$$
$$g_2(x) = \varepsilon g_{21}(x) + \varepsilon^2 g_{22}(x) + \varepsilon^3 g_{23}(x),$$
$$f(x) = \varepsilon f_1(x) + \varepsilon^2 f_2(x) + \varepsilon^3 f_3(x),$$

where g_{1i}, g_{2i}, f_i have degree k, m and n respectively for each $i = 1, 2, 3$, and ε is a small parameter. When $g_1(x) = 0$ the differential system (1) coincides with the generalized Liénard polynomial differential systems. The classical Liénard polynomial differential systems are

$$\dot{x} = y, \quad \dot{y} = -x - f(x)y, \tag{2}$$

where $f(x)$ is a polynomial in the variable x of degree n. For these systems [Lins *et al.*, 1977] stated the conjecture that if $f(x)$ has degree $n \geq 1$ then system (2) has at most $[n/2]$ limit cycles. Here $[x]$ denotes the integer part function of $x \in \mathbb{R}$. They proved this conjecture for $n = 1, 2$. The conjecture for $n = 3$ has been proved recently in [Li & Llibre, 2012]. For $n \geq 5$ the conjecture is not true, see [De Maesschalck & Dumortier, 2011; Dumortier *et al.*,