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Generalized Weierstrass integrability for the complex differential equations $\frac{dy}{dx} = a(x)y^4 + b(x)y^3 + c(x)y^2 + d(x)y + e(x)$

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ABSTRACT

We characterize the differential equations of the form

$$\frac{dy}{dx} = a(x)y^4 + b(x)y^3 + c(x)y^2 + d(x)y + e(x),$$

where a, b, c, d, e are meromorphic functions in the variable x, that admits either a generalized Weierstrass first integral or a generalized Weierstrass inverse integrating factor.

1. Introduction and statement of the main results

Let x and y be complex variables. In this paper we study the differential equations of the form

$$\frac{dy}{dx} = a(x)y^4 + b(x)y^3 + c(x)y^2 + d(x)y + e(x), \tag{1}$$

where a, b, c, d, e are meromorphic functions in the variable x. In fact, when $a(x) \equiv 0$ and $b(x) \not\equiv 0$ the differential equation (1) is an *Abel differential equation*; when $a(x) = b(x) \equiv 0$ and $c(x) \not\equiv 0$ it is a *Riccati differential equation*; when $a(x) = b(x) = c(x) \equiv 0$ and $d(x) \not\equiv 0$ it is a *linear differential equation*.

When $a(x) \equiv 0$, Eqs. (1) were studied for the first time by Abel in his analysis on the elliptic functions (see [1]). Abel equations appear in the reduction of order of many second and higher order families of differential equations. Hence they are frequently found in the modeling of real problems in several areas. Thus, for instance, Abel differential equations appear in cosmology (see [2]), in control theory of electrical circuits (see [3]), and in ecology (see [4]), ...

In what follows instead of working with the differential equation (1) we shall work with the equivalent differential system

$$\dot{x} = 1, \qquad \dot{y} = a(x)y^4 + b(x)y^3 + c(x)y^2 + d(x)y + e(x), \tag{2}$$

where the dot denotes the derivative with respect to the time t, real or complex.

The goal of this paper is to study the integrability of the differential equations (1) restricted to a special kind of first integrals. For such systems, the notion of integrability is based on the existence of the first integral, and we want to characterize when the differential equations (1) have either a generalized Weierstrass first integral or a generalized Weierstrass inverse integrating factor.

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