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Liouvillian First Integrals for Generalized Liénard Polynomial Differential Systems

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Abstract

We study the Liouvillian first integrals for the generalized Liénard polynomial differential systems of the form x' = y, y' = -g(x) - f(x)y, where g(x) and f(x) are arbitrary polynomials such that $2 \le \deg g \le \deg f$.

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1 Introduction and statement of the main result

One of the more classical problems in the qualitative theory of planar differential systems depending on parameters is to characterize the existence or not of first integrals. This is a difficult problem because in general there are no tools for solving it.

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