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Generalized Weierstrass Integrability of the Abel Differential Equations

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Abstract. We study the Abel differential equations that admit either a generalized Weierstrass first integral or a generalized Weierstrass inverse integrating factor.

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1. Introduction and statement of the main results

Let x and y be complex variables. In this paper we study the differential equations of the form

$$\frac{dy}{dx} = a(x)y^3 + b(x)y^2 + c(x)y + d(x), \tag{1.1}$$

where a, b, c, d are meromorphic functions of x. In fact, the differential equation (1.1) is called *Abel differential equation* when $a(x) \not\equiv 0$, *Riccati differential equation* when $a(x) \equiv 0$ and $b(x) \not\equiv 0$, and *linear differential equation* when $a(x) = b(x) \equiv 0$ and $c(x) \not\equiv 0$.

Equations in the form (1.1) where first studied by Abel in his analysis on the elliptic functions (see [1]). Abel equations appear in the reduction of order of many second and higher order families, and hence are frequently found in the modeling of real problems in several areas. Thus, for instance, they appear in cosmology (see [11]), in control theory of electrical circuits (see [7]), in ecology (see [6]), etc.

In what follows instead of working with the Abel differential equation (1.1) we shall work with the equivalent differential system

$$\dot{x} = 1, \quad \dot{y} = a(x)y^3 + b(x)y^2 + c(x)y + d(x), \tag{1.2}$$

where the dot denotes derivative with respect to the time t, real or complex.

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