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Darboux integrability of a generalized Friedmann-Robertson-Walker Hamiltonian system

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We study the Darboux first integrals of a generalized Friedmann-Robertson-Walker Hamiltonian system.

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1. Introduction and statement of the main result

Given a system of ordinary differential equations depending on parameters in general is very difficult to recognize for which values of the parameters the equations have first integrals because there are no satisfactory methods to answer this question.

In this paper we study the first integrals of a generalized Friedmann-Robertson-Walker Hamiltonian differential system in $\mathbb{R}^4$

$$\begin{aligned} \dot{x} &= -p_x, \\ \dot{y} &= p_y, \\ \dot{p}_x &= x - ax^3 - bxy^2, \\ \dot{p}_y &= -y - bx^2y, \end{aligned} \tag{1.1}$$

with $a, b \in \mathbb{R}$ being parameters and the dot denotes derivative with respect to time t.

The Hamiltonian of this system is

$$H_0 = \frac{1}{2}(p_y^2 - p_x^2) + \frac{1}{2}(y^2 - x^2) + \frac{a}{4}x^4 + \frac{b}{2}x^2y^2.$$

When $a = 0$, system (1.1) coincides with the Friedmann-Robertson-Walker system that modelates a universe, filled by a conformally coupled by a massive real scalar field (see [2] for more details where the authors pesent analytical and numerical evidence of the existence of chaotic