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# On the analytic integrability of the cored galactic Hamiltonian

Jaume Llibre [a,*], Claudia Valls [b]

[a] Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain
[b] Departamento de Matemática, Instituto Superior Técnico, Universidade Técnica de Lisboa, Av. Rovisco Pais 1049–001, Lisboa, Portugal

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ABSTRACT

We provide a complete characterization of the analytic first integrals of the cored galactic Hamiltonian.



## 1. Introduction and statement of the main results

We consider the planar cored galactic Hamiltonian

$$H = \frac{1}{2}(p_x^2 + p_y^2/q) + \sqrt{1 + x^2 + \frac{y^2}{q^2}}, \tag{1}$$

where $q > 0$. Its associated Hamiltonian system is

$$\begin{aligned} x' &= p_x, \\ y' &= \frac{p_y}{q}, \\ p_x' &= -\frac{x}{\sqrt{1 + x^2 + y^2/q^2}}, \\ p_y' &= -\frac{y}{q\sqrt{1 + x^2 + y^2/q^2}}, \end{aligned} \tag{2}$$

where the prime denotes derivative with respect to time $t$. Note that this Hamiltonian system has two degrees of freedom.

The potential

$$\sqrt{1 + x^2 + \frac{y^2}{q^2}}$$

has an absolute minimum and a reflection symmetry with respect to the two axes $x$ and $y$. The motivation for the choice of these symmetries comes from the interest of this potential in galactic dynamics, see for instance [1–11]. The parameter $q$

* Corresponding author. Tel.: +34 935811303; fax: +34 935812790.
*E-mail addresses:* jllibre@mat.uab.cat (J. Llibre), cvalls@math.ist.utl.pt (C. Valls).

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