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Darboux integrability of 2-dimensional Hamiltonian systems with homogenous potentials of degree 3

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We provide a characterization of all Hamiltonian systems of the form $H = (p_1^2 + p_2^2)/2 + V(q_1, q_2)$, where V is a homogenous polynomial of degree 3 which are completely integrable with Darboux first integrals. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4868701]

I. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

We consider $\mathbb{R}^4$ as a symplectic linear space with canonical variables $q = (q_1, q_2)$ and $p = (p_1, p_2)$, with q_i called the *coordinates* and p_i called the *momenta*. We want to study the Hamiltonian systems with Hamilton's function of the form

$$H = \frac{1}{2}\sum_{i=1}^{2} p_i^2 + V(q),$$

where $V(q) = V(q_1, q_2)$ is a homogeneous polynomial of degree 3, i.e., we will study the Hamiltonian systems

$$\dot{q}_i = p_i, \quad \dot{p}_i = -\frac{\partial V}{\partial q_i}, \quad i = 1, 2. \tag{1}$$

During the last century these systems have been actively investigated, with special attention to these systems where V has degree at most 5 and which have a second polynomial first integral with degree at most 4 in the variables p_1 and p_2 (see, for instance, Refs. 1, 2, 10, 11, and 19). When the homogeneous potential has degree three we want to mention the works of Hietarinta[12,13] where the author studied the polynomial integrability assuming that the additional first integral is a polynomial of order at most four in the momenta.

We start by recalling some definitions. Let $A = A(p, q)$ and $B = B(p, q)$ be two functions. Their *Poisson bracket* $\{A, B\}$ is defined as

$$\{A, B\} = \sum_{i=1}^{2}\left(\frac{\partial A}{\partial q_i}\frac{\partial B}{\partial p_i} - \frac{\partial A}{\partial p_i}\frac{\partial B}{\partial q_i}\right).$$

We say that two functions A and B are *in involution* if $\{A, B\} = 0$. We say that a non-constant function $F = F(q, p)$ is a *first integral* for the Hamiltonian system (1) if it commutes with the Hamiltonian function H, i.e., $\{H, F\} = 0$. We say that the Hamiltonian system (1) is *completely integrable* if it has 2 functionally independent first integrals which are in involution. One first integral will always be the Hamiltonian H. We say that two functions H and F are *independent* if their gradients are linearly independent at all points $\mathbb{R}^4$ except perhaps in a zero Lebesgue set.

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