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Hopf bifurcation of a generalized Moon–Rand system

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ABSTRACT

We study the Hopf bifurcation from the equilibrium point at the origin of a generalized Moon–Rand system. We prove that the Hopf bifurcation can produce 8 limit cycles. The main tool for proving these results is the averaging theory of fourth order. The computations are not difficult, but very big and have been done with the help of Mathematica and Mapple.

1. Introduction and statement of the main result

The Moon–Rand systems were developed to model the control of flexible space structures (see [8,2,6]). They were introduced by Moon and Rand. It is a differential equation in $\mathbb{R}^3$ of the form

$$\begin{aligned} \dot{u} &= v, \\ \dot{v} &= -u - uw, \\ \dot{w} &= -\lambda w + \sum_{i=0}^{2} c_i u^i v^{2-i}, \end{aligned} \tag{1}$$

where c_i are real parameters. In [6] Mahdi, Romanovski and Shafer studied the Hopf bifurcation of the equilibrium point localized at the origin of system (1) using the reduction to the center manifold and studying on this surface the Hopf bifurcation. They found that 2 limit cycles can bifurcate from the origin of system (1).

In this paper we study the Hopf bifurcation of the equilibrium point localized at the origin of the generalized Moon–Rand systems

$$\begin{aligned} \dot{u} &= v, \\ \dot{v} &= -u - uw, \\ \dot{w} &= -\lambda w + \sum_{i=0}^{2}\sum_{j=0}^{2-i} b_{ij} u^i v^j w^{2-i-j} + \sum_{i=0}^{3}\sum_{j=0}^{3-i} c_{ij} u^i v^j w^{3-i-j}, \end{aligned} \tag{2}$$

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