The Co-circular Central Configurations of the 5-Body Problem

Jaume Llibre · Claudia Valls

Abstract Chenciner in 2001 asked: *Is the regular n-gon with equal masses the unique central configuration such that all the bodies lie on a circle, and the center of mass coincides with the center of the circle*? This question has a positive answer for $n = 3$. Hampton in 2003 proved that also this question has a positive answer for $n = 4$. Here we provide a positive answer for $n = 5$.

1 Introduction

The main problem of the classical celestial mechanics is the *n-body problem*; i.e. the description of the motion of n particles of positive masses under their mutual Newtonian gravitational forces. This problem is completely solved only when $n = 2$, and for $n > 2$ there are only few partial results.

Consider the Newtonian n-body problem in the plane $\mathbb{R}^2$, i.e.

$$\ddot{\mathbf{r}}_i = \sum_{j=1,\ j\neq i}^{n} \frac{m_j(\mathbf{r}_j - \mathbf{r}_i)}{r_{ij}^3}, \quad \text{for} \quad i = 1, \ldots, n.$$

J. Llibre (✉)
Departament de Matemàtiques, Universitat Autònoma de Barcelona,
08193 Bellaterra, Barcelona, Catalonia, Spain
e-mail: jllibre@mat.uab.cat

C. Valls
Departamento de Matemática, Instituto Superior Técnico,
Universidade Técnica de Lisboa, 1049-001 Lisboa, Portugal
e-mail: cvalls@math.ist.utl.pt