



On the integrability of a three-dimensional cored galactic Hamiltonian



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ABSTRACT

We characterize when the three-dimensional cored galactic Hamiltonian system with Hamiltonian

$$H = \frac{1}{2} \left(p_x^2 + p_y^2 + \frac{p_z^2}{q} \right) + \sqrt{1 + x^2 + y^2 + \frac{z^2}{q}},$$

is completely meromorphically integrable when $q \in [0.36, 1]$. The key point for this characterization is to transform the non-polynomial cored Hamiltonian system into a polynomial one.

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1. Introduction and statement of the main results

We consider the three-dimensional cored galactic Hamiltonian

$$H = \frac{1}{2} \left(p_x^2 + p_y^2 + \frac{p_z^2}{q} \right) + \sqrt{1 + x^2 + y^2 + \frac{z^2}{q}},$$

where $q > 0$. Its associated Hamiltonian system is

$$x' = p_x,$$

$$y' = p_y,$$

$$z' = \frac{p_z}{q},$$

$$p'_x = -\frac{x}{\sqrt{1 + x^2 + y^2 + z^2/q}},$$

$$p'_y = -\frac{y}{\sqrt{1 + x^2 + y^2 + z^2/q}},$$

$$p'_z = -\frac{z}{q\sqrt{1 + x^2 + y^2 + z^2/q}},$$

(1)

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