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ANALYTIC INTEGRABILITY OF A CLASS OF PLANAR POLYNOMIAL DIFFERENTIAL SYSTEMS

Jaume Llibre

Departament de Matemàtiques, Universitat Autònoma de Barcelona
08193 Bellaterra, Barcelona, Catalonia, Spain

Claudia Valls

Departamento de Matemática, Instituto Superior Técnico
Universidade Técnica de Lisboa, Av. Rovisco Pais 1049–001
Lisboa, Portugal

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Abstract. In this paper we find necessary and sufficient conditions in order that the differential systems of the form $\dot{x} = xf(y)$, $\dot{y} = g(y)$, with f and g polynomials, have a first integral which is analytic in the variable x and meromorphic in the variable y. We also characterize their analytic first integrals in both variables x and y.

These polynomial differential systems are important because after a convenient change of variables they contain all quasi–homogeneous polynomial differential systems in $\mathbb{R}^2$.

1. **Introduction and statement of the main results.** Let $\mathbb{C}$ be the set of complex numbers and $\mathbb{C}[y]$ the ring of all polynomials in the variable y with coefficients in $\mathbb{C}$. In this paper we consider the polynomial differential systems of the form

$$\dot{x} = xf(y), \quad \dot{y} = g(y), \tag{1}$$

where $f, g \in \mathbb{C}[y]$ and are coprime. The dot denotes the derivative with respect to the independent variable t real or complex. We denote by $\mathcal{X} = (xf(y), g(y))$ the polynomial vector field associated to system (1), and we say that the degree of the system is $n = \max\{\deg\, xf(y), \deg\, g(y)\}$. For the sake of simplicity, we assume for the rest of the paper that system (1) is not linear, that is $n > 1$.

We recall that given a planar polynomial differential system (1), we say that a function $H : \mathcal{U} \subset \mathbb{C}^2 \to \mathbb{C}$ with $\mathcal{U}$ an open set, is a *first integral* of system (1) if H is continuous, not locally constant and constant on each trajectory of the system

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