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POLYNOMIAL FIRST INTEGRALS FOR WEIGHT-HOMOGENEOUS PLANAR POLYNOMIAL DIFFERENTIAL SYSTEMS OF WEIGHT DEGREE 4

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ABSTRACT. We classify all of the weight-homogeneous planar polynomial differential systems of weight degree 4 having a polynomial first integral.

1. Introduction and statement of the main result. In this paper, we deal with polynomial differential systems of the form:

$$\frac{d\mathbf{x}}{dt} = \dot{\mathbf{x}} = \mathbf{P}(\mathbf{x}), \quad \mathbf{x} = (x, y) \in \mathbb{C}^2, \tag{1.1}$$

with $\mathbf{P}(\mathbf{x}) = (P_1(\mathbf{x}), P_2(\mathbf{x}))$ and $P_i \in \mathbb{C}[x, y]$ for $i = 1, 2$. As usual, $\mathbb{Q}^+$, $\mathbb{R}$ and $\mathbb{C}$ will denote the sets of non-negative rational, real and complex numbers, respectively, and $\mathbb{C}[x, y]$ denotes the polynomial ring over $\mathbb{C}$ in the variables x, y. Here, t is real or complex.

System (1.1) is *weight homogeneous* or *quasi-homogeneous* if there exist $\mathbf{s} = (s_1, s_2) \in \mathbb{N}^2$ and $d \in \mathbb{N}$ such that, for arbitrary $\alpha \in \mathbb{R}^+ = \{a \in \mathbb{R},\ a > 0\}$,

$$P_i(\alpha^{s_1} x, \alpha^{s_2} y) = \alpha^{s_i - 1 + d} P_i(x, y), \tag{1.2}$$

for $i = 1, 2$. We call $\mathbf{s} = (s_1, s_2)$ the *weight exponent* of system (1.1) and d the *weight degree* with respect to the weight exponent $\mathbf{s}$. In the particular case where $\mathbf{s} = (1, 1)$, system (1.1) is called a *homogeneous polynomial differential system* of degree d.

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