COMPUTING POLYNOMIAL SOLUTIONS OF EQUIVARIANT POLYNOMIAL ABEL DIFFERENTIAL EQUATIONS

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ABSTRACT. Let $a(x)$ non-constant and $b_j(x)$ for $j = 0, 1, 2, 3$ be real or complex polynomials in the variable x. Then the real or complex equivariant polynomial Abel differential equations $a(x)\dot{y} = b_1(x)y + b_3(x)y^3$ with $b_3(x) \neq 0$, and the real or complex polynomial equivariant polynomial Abel differential equations of second kind $a(x)y\dot{y} = b_0(x) + b_2(x)y^2$ with $b_2(x) \neq 0$, have at most 7 polynomial solutions. Moreover there are equations of these type having these maximum number of polynomial solutions.

1. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

Abel differential equations of first kind

$$a(x)\dot{y} = b_0(x) + b_1(x)y + b_2(x)y^2 + b_3(x)y^3 \tag{1}$$

with $b_3(x) \neq 0$ appear in many text-books of ordinary differential equations as one of first non-trivial examples of nonlinear differential equations, see for instance [10]. Here the dot denotes the derivative with respect to the independent variable x. If $b_3(x) = b_0(x) = 0$ or $b_2(x) = b_0(x) = 0$ the Abel differential equation reduces to a Bernoulli differential equation, while if $b_3(x) = 0$ the Abel differential equation reduces to a Riccati differential equation.

The Abel differential equations (1) have been studied intensively, either calculating their solutions (see for instance [7, 11, 12, 13]), or classifying their centers (see [2, 3, 4]), and recently in [6, 8, 9] the authors studied the polynomial solutions of the differential equation $y' = \sum_{i=0}^{n} a_i(x)y^i$.

The analysis of particular solutions (as polynomial or rational solutions) of the differential equations is important for understanding the set of solutions of a differential equation. In 1936 Rainville [14] characterized the Riccati differential equations $\dot{y} = b_0(x) + b_1(x)y + y^2$, with $b_0(x)$ and $b_1(x)$ polynomials in the variable x, having polynomial solutions.

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