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Limit cycles for a variant of a generalized Riccati equation

Jaume Llibre[a,*], Claudia Valls[b]

[a] *Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain*

[b] *Departamento de Matemática, Instituto Superior Técnico, Universidade Técnica de Lisboa, Av. Rovisco Pais 1049-001, Lisboa, Portugal*

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A B S T R A C T

In this paper we provide a lower bound for the maximum number of limit cycles surrounding the origin of systems $(\dot{x}, \dot{y} = \ddot{x})$ given by a variant of the generalized Riccati equation

$$\ddot{x} + \varepsilon x^{2n+1}\dot{x} + bx^{4n+3} = 0,$$

where $b > 0$, $b \in \mathbb{R}$, n is a non-negative integer and ε is a small parameter. The tool for proving this result uses Abelian integrals.
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1. Introduction and statement of the main results

Some variants of the generalized Riccati equation

$$\ddot{x} + \alpha x^{2n+1}\dot{x} + x^{4n+3} = 0, \tag{1}$$

have been studied for several authors, see for instance [1,2], and the references quoted there. In the first paper the authors studied mainly the following variant of Eq. (1)

$$\ddot{x} + (2n+3)x^{2n+1}\dot{x} + x^{4n+3} + \omega^2 x = 0,$$

showing numerically that such differential equation exhibits isochronous oscillations. In the second paper the authors study the variant of Eq. (1)

$$\ddot{x} + (2n+3)x^{2n+1}\dot{x} + x^{4n+3} + \omega^2 x^{2n+1} = 0,$$

and they find the analytical expression of some particular solutions.

* Corresponding author.
E-mail addresses: jllibre@mat.uab.cat (J. Llibre), cvalls@math.ist.utl.pt (C. Valls).

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