ON THE UNIQUENESS OF ALGEBRAIC LIMIT CYCLES FOR QUADRATIC POLYNOMIAL DIFFERENTIAL SYSTEMS WITH TWO PAIRS OF EQUILIBRIUM POINTS AT INFINITY

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Abstract. Algebraic limit cycles in quadratic polynomial differential systems started to be studied in 1958, and few years later the following conjecture appeared: Quadratic polynomial differential systems have at most one algebraic limit cycle.

We prove that for a quadratic polynomial differential system having two pairs of diametrally opposite equilibrium points at infinity, has at most one algebraic limit cycle. Our result provides a partial positive answer to this conjecture.

1. Introduction and statement of the main results

Let $\mathbb{R}[x,y]$ be the ring of all real polynomials in the variables x and y. Differential systems of the form

$$\frac{dx}{dt} = \dot{x} = P(x,y), \quad \frac{dy}{dt} = \dot{y} = Q(x,y), \tag{1}$$

where $P, Q \in \mathbb{R}[x,y]$ with t real are called *real polynomial differential systems*. We say that system (1) has degree m if the maximum degree of the polynomials P and Q is m. When $m = 2$, system (1) is called *a quadratic system*.

System (1) has

$$\mathcal{X} = P(x,y)\frac{\partial}{\partial x} + Q(x,y)\frac{\partial}{\partial y} \tag{2}$$

as its *polynomial vector field*.

The algebraic curve $g(x,y) = 0$ with $g = g(x,y) \in \mathbb{R}[x,y]$ is an *invariant algebraic curve* of the $\mathcal{X}$ if for some polynomial $K \in \mathbb{R}[x,y]$, we have

$$\mathcal{X}g = P\frac{\partial g}{\partial x} + Q\frac{\partial g}{\partial y} = Kg.$$

The polynomial $K = K(x,y)$ is called *the cofactor* of g. We recall that $g = 0$ is *invariant* by $\mathcal{X}$.

We say that an invariant algebraic curve $g = 0$ is *irreducible* when g is irreducible in $\mathbb{R}[x,y]$. An isolated periodic orbit in the set of periodic orbits

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