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Darboux Polynomials, Balances and Painlevé Property

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Abstract—For a given polynomial differential system we provide different necessary conditions for the existence of Darboux polynomials using the balances of the system and the Painlevé property. As far as we know, these are the first results which relate the Darboux theory of integrability, first, to the Painlevé property and, second, to the Kovalevskaya exponents. The relation of these last two notions to the general integrability has been intensively studied over these last years.

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1. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

The Painlevé property appears in studying the general solutions of the differential equations viewed as functions of complex time. More precisely, when the solutions are single-valued on its maximum domain of analytic continuation, we say that the system has the *Painlevé property*. In other words, a differential system has the Painlevé property if its general solution has no movable critical singularities, for more details see [3]. This property imposes strong conditions that, despite the fact that it has not been proved, one believes that in this case the system is integrable. However, there is no precise algorithm to decide whether a system has the Painlevé property, and only necessary conditions can be obtained, called the *Painlevé test*. Most of the systems do not satisfy the Painlevé test, but there is a lot of information concerning the global behavior of the system that we can obtain from the local analysis around the singularities in complex time and the lack of meromorphicity can be used to prove the nonintegrability of the system with meromorphic first integrals.

For more than half a century after its development, the Painlevé theory for differential equations was considered an interesting and important, but perhaps slightly old-fashioned, part of the theory of special functions, and little attention was paid to it until the early 1980s when its relation to soliton theories was discovered. Since then there has been a huge amount of work relating the Painlevé property to different branches of differential systems such as the integrability of PDEs, the rational and polynomial integrability of ODEs etc. However, very little is known about its relation to the Darboux theory of integrability for polynomial differential systems. The main aim of this paper is to focus on the connections between the existence of Darboux polynomials, the Painlevé property and the Kovalevskaya exponents (introduced by Sophia Kowalevskaya to compute the Laurent series solutions of rigid body motion).

In order to state the main results of the paper, we consider a polynomial differential system of the form

$$\frac{dx}{dt} = \dot{x} = P(x) \quad \text{with } x = (x_1, \dots, x_n) \in \mathbb{C}^n, \tag{1.1}$$

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