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On the Darboux integrability of the logarithmic galactic potentials

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ABSTRACT

We study the logarithmic Hamiltonians $H = (p_x^2 + p_y^2)/2 + \log(1 + x^2 + y^2/q^2)^{1/2}$, which appear in the study of the galactic dynamics. We characterize all the invariant algebraic hypersurfaces and all exponential factors of the Hamiltonian system with Hamiltonian H. We prove that this Hamiltonian system is completely integrable with Darboux first integrals if and only if $q = \pm 1$.

1. Introduction and statement of the main results

The potential

$$V = \frac{1}{2}\log\left(R^2 + x^2 + \frac{y^2}{q^2}\right),$$

where $q \in \mathbb{R} \setminus \{0\}$ is called the *logarithmic potential*. It has an absolute minimum and reflection symmetry with respect to both axes. This potential is relevant in problems of galactic dynamics as a model for elliptical galaxies. More precisely, it is a model of a core embedded in a dark matter halo, with R being the core radius. Without loss of generality we can assume that $R = 1$, and the energy can take any non-negative value. The parameter q is the ellipticity of the potential, which ranges in the interval $0.6 \le q \le 1$. Lower values of q have no physical meaning and greater values of q are equivalent to reverse the role of the coordinate axes. In this paper, to make a complete and deep study of the Darboux integrability of such a potential we will consider that $q \in \mathbb{R} \setminus \{0\}$. This model has been intensively investigated from different dynamical and physical points of view by several authors, see for instance [1–5].

We consider the logarithmic Hamiltonian

$$H = \frac{1}{2}(p_x^2 + p_y^2) + \frac{1}{2}\log\left(1 + x^2 + \frac{y^2}{q^2}\right), \quad q \in \mathbb{R} \setminus \{0\},$$

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