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Periodic orbits of the planar anisotropic generalized Kepler problem

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ABSTRACT

Many generalizations of the Kepler problem with homogeneous potential of degree −1/2 have been considered. Here, we deal with the generalized anisotropic Kepler problem with homogeneous potential of degree −1. We provide the explicit solutions of this problem on the zero energy level and show that all of them are periodic.

I. INTRODUCTION AND STATEMENT OF THE MAIN RESULT

The classical Kepler problem describes the motion of the two-body problem under the mutual gravitational attraction given by the Newtonian's universal law of gravitation.

In Refs. 2, 9–11, 13, 14, and 16, different generalizations of the Kepler problem with homogeneous potential of degree −1/2 have been studied, for instance, generalizations to n-dimensional curved spaces, to charge quantization, to Euclidean Jordan algebra, and to their integrability with Clifford algebras or with Lie algebras in quantum mechanics.

In Refs. 5–8, Gutzwiller generalized the Kepler problem to describe the motion of two-body in an anisotropic configuration plane with homogeneous potential of degree −1/2. Gutzwiller research wanted to find an approximation of the quantum mechanical energy levels for a chaotic system. Recently in Refs. 1, 3, and 15, some dynamics and periodic orbits of this anisotropic Kepler problem were studied analytically.

Here, we generalize the anisotropic Kepler problem from homogeneous potential of degree −1/2 to homogeneous potential of degree −1. More precisely, the equations of motion of the planar anisotropic Kepler problem with homogeneous potential of degree −1 in Hamiltonian formulation are described by the Hamiltonian,

$$H = H(x, y, p_x, p_y) = \frac{1}{2}(p_x^2 + p_y^2) - \frac{1}{(1+\varepsilon)x^2 + y^2}, \tag{1}$$

with $|\varepsilon| > 0$ being a small parameter which provides the anisotropy in the direction of the x-axis.

Note that the angular momentum for system (1) is not a first integral due to the fact that the anisotropy of the plane destroys the rotational invariance.

Our main result is the following one:

Theorem 1. *We consider the generalized anisotropic Kepler problem with homogeneous potential of degree −1 given by Hamiltonian (1). Then,*

(a) *The energy level $H = 0$ is diffeomorphic to the manifold $\mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{R}$.*