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ORIGINAL ARTICLE

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Global dynamics of the integrable Armbruster-Guckenheimer-Kim galactic potential

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Abstract We study the global dynamics of the completely integrable Armbruster-Guckenheimer-Kim galactic potential. In these cases this system has two first integrals H_1 and H_2 independent and in involution. Let I_{h_1} and I_{h_2} be the set of points of the phase space on which H_1 and H_2 take the values h_1 and h_2, respectively. The sets $I_{h_1h_2} = I_{h_1} \cap I_{h_2}$ are invariant by the dynamics. We characterize the global flow on these sets and we describe the foliation of the phase space by the invariant sets $I_{h_1h_2}$.

Keywords Armbruster-Guckenheimer-Kim galactic potential · Invariant manifolds · Complete integrability

1 Introduction

The Armbruster-Guckenheimer-Kim potential is a galactic potential introduced in Armbruster et al. (1989) that studies the dynamics for the interchanging of nearly nondegenerate modes with square symmetry. They derived the model starting with a normal form given by a system of differential equations which represented the codimension two bifurcation problem. More precisely, the Hamiltonian function that they provided is

$$H(x, p_x, y, p_y) = \frac{1}{2}\left(p_x^2 + p_y^2\right) + \frac{1}{2}\left(x^2 + y^2\right) - \frac{a}{4}\left(x^2 + y^2\right)^2 - \frac{b}{2}x^2y^2,$$

where a, b are arbitrary constants. If we add the term $-\omega(xp_y - yp_x)$ then the system describes the dynamics of rotation of a nearly axisymetric galaxy rotating with a constant velocity ω around a fixed axis. The existence of such ω denotes that the rotation of the galaxy must be taken into account when we study the stellar orbits (see Zeeuw and Merritt 1983). Many studies concerning the integrability and non-integrability of such systems have been done (see for instance Acosta-Humánez et al. 2018; Elmandouh 2016; El-Sabaa et al. 2019) using different techniques such as the Painlevé analysis and the Morales-Ramis theory as well as the study of the existence of periodic orbits which was done in Llibre and Roberto (2012). In particular, it was proved in El-Sabaa et al. (2019) that if $b = 2a$ or $b = -a$ the system is completely integrable but the authors do not describe completely the dynamics of the integrable systems form the point of view of the Liouville-Arnold theorem (see Sect. 2). This is the main aim of this paper.

When $b = 2a$ the Hamiltonian has the form

$$H = \frac{1}{2}\left(p_x^2 + p_y^2\right) + \frac{1}{2}\left(x^2 + y^2\right) - \frac{a}{4}\left(x^2 + y^2\right)^2 - ax^2y^2.$$

Introducing the new variables

$$u = \frac{1}{\sqrt{2}}(x - y), \qquad v = \frac{1}{\sqrt{2}}(x + y),$$

$$p_u = \frac{1}{\sqrt{2}}(p_x - p_y), \qquad p_v = \frac{1}{\sqrt{2}}(p_x + p_y),$$

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