ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 49, Number 2, 2019

INTEGRABILITY OF REVERSIBLE AND EQUIVARIANT QUADRATIC POLYNOMIAL DIFFERENTIAL SYSTEMS IN THE PLANE

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ABSTRACT. We study the existence of first integrals for the class of reversible and equivariant quadratic polynomial differential systems in the plane. We put special emphasis in the study of the analytic first integrals.

1. Introduction. For a planar differential system, the existence of a first integral completely determines its phase portrait. However, the explicit computation of first integrals is not an easy task. A first step is to compute those first integrals in different classes of functions such as the class of analytic functions.

Recently, quadratic vector fields have been intensively investigated as one of the easiest families of nonlinear differential systems, and more than 1,000 papers have been published about these vector fields (see, for instance, **[2, 8, 9, 10]**), but the problem of classifying all of the integrable quadratic vector fields remains open. For more information on integrable differential vector fields in dimension 2, see **[3]**, for instance.

The reversible and the equivariant differential equations have symmetries, and this is important since symmetry of a differential equation is a transformation that sends solutions to solutions simplifying the study of these differential systems. The equations describing a physical or a biological system often exhibit symmetries; therefore, it is important to

2010 AMS *Mathematics subject classification.* Primary 34C07, 34C08, 37J35.

Keywords and phrases. Reversible, equivariant, quadratic planar polynomial equations, analytic first integral.

The first author was partially supported by the Ministerio de Economía, Industria y Competitividad, Agencia Estatal de Investigación, grant No. MTM2016-77278-P (FEDER), the Agència de Gestió d'Ajuts Universitaris i de Recerca, grant No. 2017SGR1617 and the H2020 European Research Council, grant No. MSCA-RISE-2017-777911. The second author was partially supported by FCT/Portugal through UID/MAT/04459/2013.

Received by the editors on July 10, 2014, and in revised form on September 26, 2018.

DOI:10.1216/RMJ-2019-49-2-579