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On the Singularities of the Planar Cubic Polynomial Differential Systems and the Euler Jacobi Formula

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Abstract
Using the Euler–Jacobi formula we obtain an algebraic relation between the singular points of a polynomial vector field and their topological indices. Using this formula we obtain the configuration of the singular points together with their topological indices for the planar cubic polynomial differential systems when these systems have nine finite singular points.

Keywords Euler–Jacobi formula · Singular points · Topological index · Cubic polynomial differential systems

Mathematics Subject Classification Primary 34A05; Secondary 34C05 · 37C10

1 Introduction and statement of the main results

Consider in $\mathbb{R}^2$ the polynomial differential system

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y), \tag{1}$$

where $P(x, y)$ and $Q(x, y)$ are real polynomials of degree 3, called a planar cubic polynomial differential system, or simply *cubic system*.

The motivation of our paper comes from the fact that for the planar quadratic polynomial differential systems the characterization of all configurations of the indices

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