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Twin Polynomial Vector Fields of Arbitrary Degree

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Abstract
In this paper we study polynomial vector fields on $\mathbb{C}^2$ of degree larger than 2 with n^2 isolated singularities. More precisely, we show that if two polynomial vector fields share n^2-1 singularities with the same spectra (trace and determinant) and from these singularities n^2-2 have the same positions, then both vector fields have identical position and spectra at all the singularities. Moreover we also show that if two polynomial vector fields share n^2-1 singularities with the same positions and from these singularities n^2-2 have the same spectra, then both vector fields have identical position and spectra at all the singularities. Moreover we also prove that generic vector fields of degree $n>2$ have no twins and that for any $n>2$ there exist two uniparametric families of twin vector fields, i.e. two different families of vector fields having exactly the same singular points and for each singular point both vector fields have the same spectrum.

Keywords Euler–Jacobi formula · Singular points · Topological index · Polynomial differential systems · Berlinskii's Theorem

Mathematics Subject Classification Primary 34A05; Secondary 34C05 · 37C10

1 Introduction and Statement of the Main Results

Consider the polynomial vector fields on the affine plane $\mathbb{C}^2$ and denote by $\mathbb{P}_n$ the space of all polynomial vector fields

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