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# Phase Portraits of Uniform Isochronous Centers with Homogeneous Nonlinearities

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**Abstract**
We classify the phase portraits in the Poincaré disc of the differential equations of the form $x' = -y + xf(x, y)$, $\dot{y} = x + yf(x, y)$ where $f(x, y)$ is a homogeneous polynomial of degree $n - 1$ when $n = 2, 3, 4, 5$, and $f$ has only simple zeroes. We also provide some general results on these uniform isochronous centers for all $n \geq 2$. All our results have been revised by the program P4; see Chaps. 9 and 10 of Dumortier et al. (UniversiText, Springer-Verlag, New York, 2006).

**Keywords** Polynomial vector field · Uniform isochronous center · Phase portrait · Poincaré disc

**Mathematics Subject Classification (2010)** Primary 34A05 · Secondary 34C05, 37C10

## 1 Introduction and Statement of the Main Results

The firstinvestigation in isochronicity goes back to Huygens in [7] with the studyof the cycloidal pendulum in the seventeenth century. Nowadays, isochronicity appears in many physical problems and it is closely related to the existence and uniqueness of solutions for certain bifurcation problems or boundary value problems (see for instance [9] and the references therein). In the last decade, the study of the isochronicity has been growing specially in the case of polynomial differential systems due to the appearance of powerful methods of computational analysis; see for instance [1, 3, 5, 14] to cite just few of them.

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