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Global phase portraits of the generalized van der Pol systems

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ABSTRACT

We consider the generalized van der Pol systems

$$\dot{x} = y, \quad \dot{y} = -x + (1 - x^2)f(y),$$

where $f \in \mathbb{R}[y]$. The classical van der Pol systems have $f(y) = y$. We first characterize when the origin of the generalized van der Pol systems is a center, and second we provide the global phase portraits in the Poincaré disc of the generalized van der Pol when $f(y) = a_1 y + a_2 y^2$ for all $a_1, a_2 \in \mathbb{R}$.

1. Introduction and statement of the main results

In this paper we deal with the generalized van der Pol systems

$$\dot{x} = y, \quad \dot{y} = -x + (1 - x^2)f(y), \tag{1}$$

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