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The Completely Integrable Differential Systems are Essentially Linear Differential Systems

Jaume Llibre · Claudia Valls · Xiang Zhang

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Abstract Let $\dot{x} = f(x)$ be a C^k autonomous differential system with $k \in \mathbb{N} \cup \{\infty, \omega\}$ defined in an open subset Ω of $\mathbb{R}^n$. Assume that the system $\dot{x} = f(x)$ is C^r completely integrable, i.e., there exist $n-1$ functionally independent first integrals of class C^r with $2 \le r \le k$. As we shall see, we can assume without loss of generality that the divergence of the system $\dot{x} = f(x)$ is not zero in a full Lebesgue measure subset of Ω. Then, any Jacobian multiplier is functionally independent of the $n-1$ first integrals. Moreover, the system $\dot{x} = f(x)$ is C^{r-1} orbitally equivalent to the linear differential system $\dot{y} = y$ in a full Lebesgue measure subset of Ω. Additionally, for integrable polynomial differential systems, we characterize their type of Jacobian multipliers.

Keywords Differential systems · Completely integrability · Orbital equivalence · Normal form · Jacobian multiplier · Polynomial differential systems

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J. Llibre
Departament de Matemàtiques, Universitat Autònoma de Barcelona, Bellaterra, 08193 Barcelona, Catalonia, Spain
e-mail: jllibre@mat.uab.cat

C. Valls (✉)
Departamento de Matemática, Instituto Superior Técnico, Universidade Técnica de Lisboa, Av. Rovisco Pais, 1049-001 Lisbon, Portugal
e-mail: cvalls@math.ist.utl.pt

X. Zhang
Department of Mathematics, MOE–LSC, Shanghai Jiao Tong University, Shanghai 200240, People's Republic of China
e-mail: xzhang@sjtu.edu.cn