

Liouvillian Integrability Versus Darboux Polynomials

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Abstract In this note we provide a sufficient condition on the existence of Darboux polynomials of polynomial differential systems via existence of Jacobian multiplier or of Liouvillian first integral and a degree condition among different components of the system. As an application of our main results we prove that the Liénard polynomial differential system $\dot{x} = y$, $\dot{y} = -f(x)y - g(x)$ with $\deg f > \deg g$ is not Liouvillian integrable.

Keywords Polynomial differential system · Liouvillian integrability · Darboux Jacobian multiplier · Darboux polynomial

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