Nonlinear Dyn (2014) 78:969–980
DOI 10.1007/s11071-014-1490-2

ORIGINAL PAPER

New 1:1:1 periodic solutions in 3-dimensional galactic-type Hamiltonian systems

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Received: 24 December 2013 / Accepted: 22 May 2014 / Published online: 14 June 2014
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Abstract Applying the averaging theory, we prove the existence of new families of periodic orbits for 3-dimensional type-galactic Hamiltonian systems.

Keywords Periodic orbits · Galactic-type Hamiltonian · Averaging theory

Mathematics Subject Classification Primary 34C25 · 70F15

1 Introduction and statements of main results

In this work, we prove analytically the existence of families of periodic solutions for galactic Hamiltonian systems with 3 degrees of freedom. For a good introduction to the galactic-type Hamiltonians here studied, the reader can look at the paper of Caranicolas [4] and at the references therein for a detailed deduction and implications about the importance of these Hamiltonians.

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Let

$$H = H(q_1, q_2, q_3, p_1, p_2, p_3) = \frac{1}{2}\left(p_1^2 + p_2^2 + p_3^2\right) + \frac{1}{2}\left(q_1^2 + q_2^2 + q_3^2\right) + H_1 \tag{1}$$

be a Hamiltonian in resonance $1:1:1$, where H_1 will have two different expressions. First, we consider

$$H_1 = H_1(q_1, q_2, q_3, p_1, p_2, p_3) = -\left(q_1^2 q_2^2 + q_1^2 q_3^2 + q_2^2 q_3^2\right), \tag{2}$$

with $q_1, q_2, q_3, p_1, p_2, p_3 \in \mathbb{R}$. As Caranicolas said, the Hamiltonian (1)–(2) have been used by several authors (see for instance, Van der Aa and Sanders [12]; Martinet and Magnenat [8]; Magnenat [7]; Martinet et al [9]; Hayli et al. [5]) for studying the local dynamical properties of galaxies. Other models for studying the motion in non-axially symmetric galaxies and disk galaxies with non-spherical nuclei have been considered in [18] and [19], respectively.

The Hamiltonian systems associated to (1)–(2) are

$$\begin{aligned}
\dot{q_1} &= p_1,\\
\dot{q}_2 &= p_2,\\
\dot{q}_3 &= p_3,\\
\dot{p}_1 &= -q_1 + 2q_1\left(q_2^2 + q_3^2\right),\\
\dot{p}_2 &= -q_2 + 2q_2\left(q_1^2 + q_3^2\right),\\
\dot{p}_3 &= -q_3 + 2q_3\left(q_1^2 + q_2^2\right).
\end{aligned} \tag{3}$$