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Periodic Solutions of a Periodic FitzHugh–Nagumo System

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Recently some interest has appeared for the periodic FitzHugh–Nagumo differential systems. Here, we provide sufficient conditions for the existence of periodic solutions in such differential systems.

Keywords: FitzHugh–Nagumo system; periodic orbit; averaging theory.

1. Introduction and Statements of the Main Result

The FitzHugh–Nagumo (or FHN) differential systems are simplified models of the Hodgkin–Huxley differential system which is a simpler mathematical model for studying the nerve membrane [FitzHugh, 1961], see for more details the articles [Hodgkin & Huxley, 1952; Nagumo *et al.*, 1962; Murray, 1989; Rinzel, 1981] and references therein.

In this work, we study the existence of periodic solutions of the following periodic FitzHugh–Nagumo differential system

$$\begin{aligned}\frac{dv}{dt} &= \dot{v} = -v(v-1)(v-b(t)) - w + I(t),\\ \frac{dw}{dt} &= \dot{w} = av - cw.\end{aligned} \tag{1}$$

Here v is the analog of the nerve membrane potential, w represents ion concentrations; I is an applied current (stimulus current). This model (1) can be considered as an extension of the FHN model, because it includes a time-varying threshold given by $b(t)$ which corresponds, for example, to the threshold between electrical silence and electrical firing (see details in [Faghih *et al.*, 2010; Brown *et al.*, 2001] and references therein). Also this model includes a periodic forcing $I(t)$. Periodical forcing has been considered in [Kostova *et al.*, 2004]. In this paper, both functions $b(t)$ and $I(t)$ are T-periodic, i.e. $b(t+T) = b(t)$ and $I(t+T) = I(t)$ for all $t \in \mathbb{R}$.

The system with I and b constant has been considered by several authors, for example, in [Kalachev, 1993] using techniques from the ordinary differential equations, and the possibility is analyzed for obtaining relaxation wave solutions and also the asymptotic solution having the structure of a relaxation wave.

The system (1) with $I(t)$ periodic and $b(t)$ constant was considered in [Chou & Lin, 1996]. A numerical study using the Poincaré map is done, and in particular, the authors analyzed the consequences of imposing a sinusoidal perturbation of the form $I_0 + I\cos\gamma t$ on the base current. A similar