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HOPF PERIODIC ORBITS FOR A RATIO–DEPENDENT PREDATOR–PREY MODEL WITH STAGE STRUCTURE

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Abstract. A ratio–dependent predator-prey model with stage structure for prey was investigated in [8]. There the authors mentioned that they were unable to show if such a model admits limit cycles when the unique equilibrium point E^* at the positive octant is unstable.

Here we characterize the existence of Hopf bifurcations for the systems. In particular we provide a positive answer to the above question showing for such models the existence of small–amplitude Hopf limit cycles being the equilibrium point E^* unstable.

1. **Introduction and statement of the main results.** Frequently it is assumed in the ratio–dependent predator-prey models that each individual prey admits the same risk to be attacked by a predator. This hypothesis is not always realistic for many species. Thus there are many species whose individuals along his life pass through two stages, immature and mature. Here the prey individuals are classified in immature or mature, and we assume that the immature ones cannot be attacked by the predators. This assumption is reasonable for many mammals, because the immature preys concealed in a mountain cave, are raised by their parents and they do not necessarily go out for seeking food, so the possibility of being attacked by the predators is negligible.

Stage structured models have been studied with attention in these last years. Thus a stage–structured model of single species growth with of immature and mature individuals was stated and analyzed in [1]. Later on in [2] it was also assumed that the time from immaturity to maturity is itself state dependent. More recently, in the articles [4, 5, 6, 7, 9] the authors considered predator–prey models with stage structure for prey or predator in order to analyze the influence of a stage structure for the prey or the predator. Xu, Chaplain and Davidson [8] studied the effect of

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