Differential and Integral Equations Volume 11, Number 1, January 1998, pp. 191 – 199

A POINCARE INDEX FORMULA FOR SURFACES WITH BOUNDARY

JAUME LLIBRE AND JORDI VILLADELPRAT
Departament de Matemàtiques, Universitat Autònoma de Barcelona
Bellaterra, 08193 Barcelona, Spain

(Submitted by: Jean Mawhin)

Abstract. We give an easy extension of the Poincaré Index Formula from the disc to any surface with boundary. In particular we show that the sum of the indices of the vector field at the critical points depends only on the Euler characteristic of the surface and on the behaviour of its trajectories in the boundary. Our theorem improves previous results on the same formula.

1. Introduction and statement of the main result. Let M be a smooth compact manifold with boundary ∂M. Let X be a continuous tangent vector field on M vanishing nowhere on ∂M. The topological index of X, $I(X)$, is the intersection number of $X(M)$ and the zero section $Z(M)$ in TM. $I(X)$ is an integer, constant under continuous variation of X through such fields vanishing nowhere on ∂M. If X has finitely many critical points $p_1, p_2, \ldots, p_k$ then it follows that

$$I(X) = \sum_{j=1}^{k} i_X(p_j),$$

where $i_X(p_j)$ denotes the index of X at the critical point p_j, $j = 1, 2, \ldots, k$. For more details, see [3] or [5].

A well-known fact is (see for instance [4]):

Poincaré Index Formula for the disc. *Let X be a continuous vector field on the closed disc $\mathbb{D}$ with finitely many critical points $p_1, p_2, \ldots, p_k$, all of them contained in the interior of $\mathbb{D}$. Assume that the number of tangency points is finite. Then*

$$\sum_{j=1}^{k} i_X(p_j) = 1 + \frac{i - e}{2},$$

where e and i denote respectively the number of exterior and interior tangencies.

For a definition of exterior and interior tangency see the remark of Section 2. Another well-known fact is the following result (see [5] or [7]).

Received for publication October 1996.
AMS Subject Classifications: 34C35, 58F.