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PLANAR CUBIC POLYNOMIAL DIFFERENTIAL SYSTEMS WITH THE MAXIMUM NUMBER OF INVARIANT STRAIGHT LINES

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ABSTRACT. We classify all cubic systems possessing the maximum number of invariant straight lines (real or complex) taking into account their multiplicities. We prove that there are exactly 23 topological different classes of such systems. For every class we provide the configuration of its invariant straight lines in the Poincaré disc. Moreover, every class is characterized by a set of affine invariant conditions.

1. Introduction and statement of the main results. We consider here the real polynomial differential system

$$\frac{dx}{dt} = P(x,y), \qquad \frac{dy}{dt} = Q(x,y), \tag{1}$$

where P, Q are polynomials in x, y with real coefficients, i.e., P, $Q \in \mathbf{R}[x,y]$. We shall say that system (1) is *cubic* if $\max(\deg(P), \deg(Q)) = 3$.

A straight line $ux + vy + w = 0$ satisfies

$$u\,\frac{dx}{dt} + v\,\frac{dy}{dt} = uP(x,y) + vQ(x,y) = (ux + vy + w)\,R(x,y)$$

for some polynomial $R(x,y)$ if and only if it is *invariant* under the flow of the system. If some of the coefficients u, v, w of an invariant straight line belong to $\mathcal{C} \setminus \mathbf{R}$, then we say that *the straight line is complex*; otherwise *the straight line is real.* Note that, since system (1) is real, if it has a complex invariant straight line $ux + vy + w = 0$, then it also has its conjugate complex invariant straight line $\bar{u}x + \bar{v}y + \bar{w} = 0$.

Let

$$\mathbf{X} = P(x,y)\,\frac{\partial}{\partial x} + Q(x,y)\,\frac{\partial}{\partial y}$$

be the polynomial vector field corresponding to system (1).

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