QUALITATIVE BEHAVIOUR OF THE FLOW OF THE n-BODY PROBLEM IN THE ZERO ENERGY LEVEL

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ABSTRACT

In this note we summarize some recent techniques and results on the singularities in the equations of motion of a class of classical mechanical systems. We use these geometric techniques to give a characterization of the global behaviour of the solutions of the flow on the zero energy level for this class of classical mechanical systems which contain the gravitational n-body problem.

1. Introduction

Newton's equation may be written

$$\dot{q} = M^{-1}p, \quad \dot{p} = -\nabla V(q), \tag{1}$$

with $q = (q_1, \ldots, q_n)^t$ and $p = (p_1, \ldots, p_n)^t$, where q_i, $p_i \in \mathbb{R}^d$ are the position and momentum of the body with mass m_i. Here q is a point in Q, an open subset of $\mathbb{R}^{nd}$, and the *potential energy function* $V : Q \to \mathbb{R}$ is real analytic, homogeneous of degree $-k$ (i.e., $V(rq) = r^{-k}V(q)$ for all $r \in \mathbb{R}$), and has an isolated singularity at the origin. The mass matrix M is a diagonal matrix with positive entries $m_1, \ldots, m_1, \ldots, m_n, \ldots, m_n$. Each mass appears exactly d times. Q is called the *configuration space* and $Q \times \mathbb{R}^{nd}$, the *phase space* of system (1). On the phase space our system is a first order system of differential equations or a vector field.

System (1) may be written in Hamiltonian form by introducing the *Hamiltonian* or *total energy function*

$$H(q,p) = \frac{1}{2}p^t M^{-1} p + V(q). \tag{2}$$