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Local Darboux first integrals of analytic differential systems

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Abstract

We discuss local and formal Darboux first integrals of analytic differential systems, using the theory of Poincaré–Dulac normal forms, and we study the effect of local Darboux integrability on analytic normalization. Moreover we determine local restrictions on classical Darboux integrability of polynomial systems.

Keywords: Polynomial differential system; Analytic differential system; Local Darboux first integral; Poincaré–Dulac normal form

1. Introduction and statement of the main results

In the present paper we will discuss an extension of Darboux integrability to (germs of) complex local analytic vector fields, and to formal vector fields on $\mathbb{C}^n$. Classically, Darboux integrability was introduced for planar polynomial vector fields [8,22,6], but the notion, which will be recalled for the reader's convenience, directly carries over to polynomial vector fields in higher dimension [9,11,19,14–16]. Thus, one says that a differential equation

$$\dot{x} = P(x) := \begin{pmatrix} P_1(x) \\ \vdots \\ P_n(x) \end{pmatrix} \tag{1}$$

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