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Hopf bifurcation for degenerate singular points of multiplicity $2n-1$ in dimension 3

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Abstract

The main purpose of this paper is to study the Hopf bifurcation for a class of degenerate singular points of multiplicity $2n-1$ in dimension 3 via averaging theory. More specifically, we consider the system

$$\begin{aligned}\dot{x} &= -H_y(x,y) + P_{2n}(x,y,z) + \varepsilon P_{2n-1}(x,y),\\ \dot{y} &= H_x(x,y) + Q_{2n}(x,y,z) + \varepsilon Q_{2n-1}(x,y),\\ \dot{z} &= R_{2n}(x,y,z) + \varepsilon c z^{2n-1},\end{aligned}$$

where

$$\begin{aligned}H &= \frac{1}{2n}\big(x^{2l}+y^{2l}\big)^m, \quad n = lm,\\ P_{2n-1} &= x\big(p_1x^{2n-2} + p_2x^{2n-3}y + \cdots + p_{2n-1}y^{2n-2}\big),\\ Q_{2n-1} &= y\big(p_1x^{2n-2} + p_2x^{2n-3}y + \cdots + p_{2n-1}y^{2n-2}\big),\end{aligned}$$

and P_{2n}, Q_{2n} and R_{2n} are arbitrary analytic functions starting with terms of degree $2n$. We prove using the averaging theory of first order that, moving the parameter ε from $\varepsilon = 0$ to $\varepsilon \neq 0$ sufficiently small, from the origin it can bifurcate $2n-1$ limit cycles, and that using the averaging theory of second order from the origin it can bifurcate $3n-1$ limit cycles when $l = 1$.

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