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Linear estimate for the number of limit cycles of a perturbed cubic polynomial differential system

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Abstract

Perturbing the cubic polynomial differential systems $\dot{x} = -y(a_1x+a_0)(b_1y+b_0)$, $\dot{y} = x(a_1x+a_0)(b_1y+b_0)$ having a center at the origin inside the class of all polynomial differential systems of degree n, we obtain using the averaging theory of second order that at most $17n + 15$ limit cycles can bifurcate from the periodic orbits of the center.

MSC: 37G15; 37D45

Keywords: Limit cycle; Averaging theory; Polynomial differential system

1. Introduction

In this paper we consider the following planar differential equations:

$$\begin{aligned} x' &= -y(a_1x + a_0)(b_1y + b_0) + \varepsilon p(x, y), \\ y' &= x(a_1x + a_0)(b_1y + b_0) + \varepsilon q(x, y), \end{aligned} \tag{1}$$

where $a_i, b_i \neq 0$ for $i = 1, 2$, ε is small enough, p and q are two arbitrary polynomials of degree n. The systems have essentially a linear center with two straight lines of singular points when $\varepsilon = 0$.

The distribution of the limit cycles is one of the basic problems in the qualitative theory of real plane differential systems. A classical way to obtain limit cycles is perturbing the periodic orbits of a center, for example perturbing the linear center $\dot{x} = -y$, $\dot{y} = x$, inside the class of polynomial differential systems of degree n; i.e. $\dot{x} = -y + \varepsilon p(x, y)$, $\dot{y} = x + \varepsilon q(x, y)$, where $p(x, y)$ and $q(x, y)$ are arbitrary polynomials of degree at most n. In this case it is well known that at most $[(n - 1)/2]$ limit cycles can bifurcate from the periodic orbits of the linear center up to first order in ε; see for instance [2]. Here $[\cdot]$ denotes the integer part function.

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