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GLOBAL DYNAMICS OF A LOTKA–VOLTERRA MODEL WITH TWO PREDATORS COMPETING FOR ONE PREY*

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Abstract. In this paper, we study the global dynamics of a class of 3-dimensional Lotka–Volterra systems with eight parameters. This system describes two predators competing for the same food; i.e., they share one prey. By theoretical analysis on this system, we obtain sufficient and necessary conditions for the principle of competitive exclusion to hold and give the global dynamical behavior of the three species in the first octant. It is shown that there are two coexistence states for the three species: periodic oscillations and steady states, which depend on the resource for the prey. These results are biologically important in pest control.

Key words. predator–prey, Lotka–Volterra model, global dynamics, extinction, coexistence

AMS subject classifications. Primary, 37N25, 34C12; Secondary, 34C28, 37G20

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1. Introduction.

The study of the dynamics of predator–prey systems originated in the works of Lotka [17] and Volterra [25], who considered a model for one predator and one single prey in a constant and uniform environment, known now as the *standard Lotka–Volterra model*. In this model the populations of predator and prey permanently oscillate for almost all positive initial conditions. In the same work, Volterra also argued that the coexistence of two or more predators competing for fewer prey resources is impossible; this claim was called the *principle of competitive exclusion* (see [1], [4], [7], [19], [24]).

Hsu and Hubbell in [9] studied 4-dimensional Lotka–Volterra models with two competing predators sharing two prey species under the assumption that prey species were capable of self-reproduction and regenerated logistically in the absence of consumption, and they obtained some conditions under which the competing predators survive or die out. Their results revealed that the principle of competitive exclusion was true for some parameter values in the Lotka–Volterra model with two competing predators sharing two prey species. Note that the predator functional response to the prey density is linear in the Lotka–Volterra model. In the case when the predator functional response to the prey density is nonlinear, the principle of competitive exclusion was re-examined by Koch in [13] via numerical simulation. This showed that the coexistence of two predators competing for a single prey species was in fact possible when the predator functional response to the prey density was assumed according to Michaelis–Menten kinetics, and such coexistence occurred along what appeared to be a periodic orbit in the positive octant of $\mathbb{R}^3$ rather than an equilibrium.

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