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LIMIT CYCLES BIFURCATING FROM A NON-ISOLATED ZERO-HOPF EQUILIBRIUM OF THREE-DIMENSIONAL DIFFERENTIAL SYSTEMS

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Abstract. In this paper we study the limit cycles bifurcating from a non-isolated zero-Hopf equilibrium of a differential system in $\mathbb{R}^3$. The unfolding of the vector fields with a non-isolated zero-Hopf equilibrium is a family with at least three parameters. By using analysis techniques and the averaging theory of the second order, explicit conditions are given for the existence of one or two limit cycles bifurcating from such a zero-Hopf equilibrium. This result is applied to study three-dimensional generalized Lotka-Volterra systems in a paper by Bobieński and Żołądek (2005). The necessary and sufficient conditions for the existence of a non-isolated zero-Hopf equilibrium of this system are given, and it is shown that two limit cycles can be bifurcated from the non-isolated zero-Hopf equilibrium under a general small perturbation of three-dimensional generalized Lotka-Volterra systems.

1. Introduction

Zero-Hopf equilibrium is an equilibrium point of three-dimensional autonomous differential systems which has a zero eigenvalue and a pair of purely imaginary eigenvalues. Usually the zero-Hopf bifurcation is a two-parameter unfolding (or family) of three-dimensional autonomous differential systems with a zero-Hopf equilibrium. The unfolding has an isolated equilibrium with a zero eigenvalue and a pair of purely imaginary eigenvalues if the two parameters take zero values, and the unfolding has complex dynamics in the small neighborhood of this isolated equilibrium as the two parameters vary in a small neighborhood of the origin. This zero-Hopf bifurcation has been studied by Guckenheimer and Holmes [8, 9], Scheurle and Marsden [23], and Kuznetsov [14], and the references therein. It has been shown that some complicated invariant sets of the unfolding could be bifurcated from the isolated zero-Hopf equilibrium under some conditions. Hence, zero-Hopf bifurcation could imply a local birth of "chaos" (cf. [5, 23]). Recently there has been some theoretical analysis and numerical simulations which show that three-dimensional or four-dimensional generalized Lotka-Volterra systems allow complicated dynamics such as chaotic behavior (cf. [1, 6, 25, 27] and the references therein). The question

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