



Dynamics, integrability and topology for some classes of Kolmogorov Hamiltonian systems in $\mathbb{R}_+^4$

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Abstract

In this paper we first give the sufficient and necessary conditions in order that two classes of polynomial Kolmogorov systems in $\mathbb{R}_+^4$ are Hamiltonian systems. Then we study the integrability of these Hamiltonian systems in the Liouville sense. Finally, we investigate the global dynamics of the completely integrable Lotka–Volterra Hamiltonian systems in $\mathbb{R}_+^4$. As an application of the invariant subsets of these systems, we obtain topological classifications of the 3-submanifolds in $\mathbb{R}_+^4$ defined by the hypersurfaces $axy + bzx + cx^2y + dxy^2 + ez^2w + fzw^2 = h$, where a, b, c, d, e, f, w and h are real constants.

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1. Introduction and statement of the main results

Kolmogorov systems are defined by the system of ordinary differential equations

$$\frac{dx_i(t)}{dt} = x_i(t) P_i(x_1(t), \dots, x_n(t)), \quad i = 1, \dots, n \quad (1)$$

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